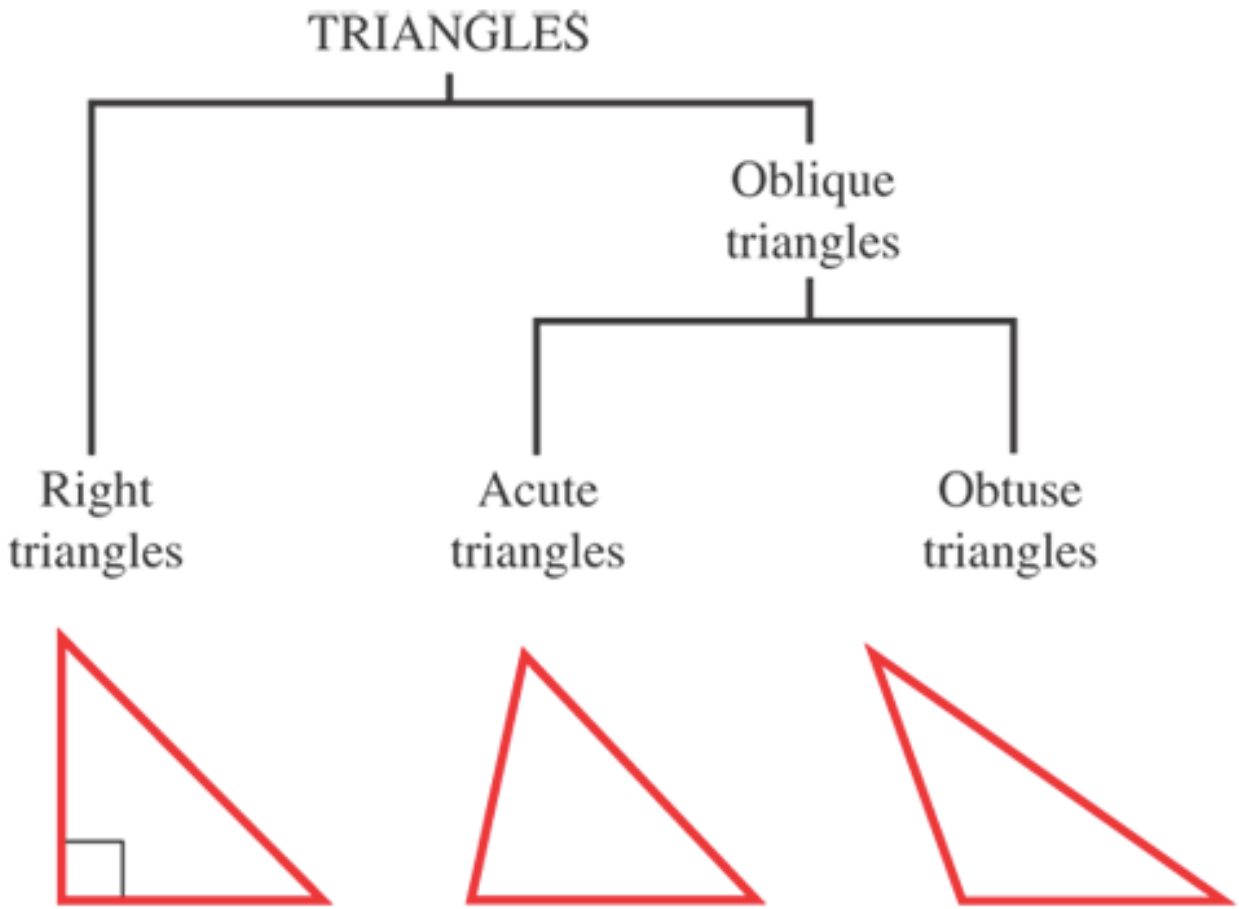
Chapter 7
Applications of Trigonometry

1. **The Law of Sines**
2. The Law of Cosines
3. The Area of a Triangle
4. Vectors
5. The Dot Product

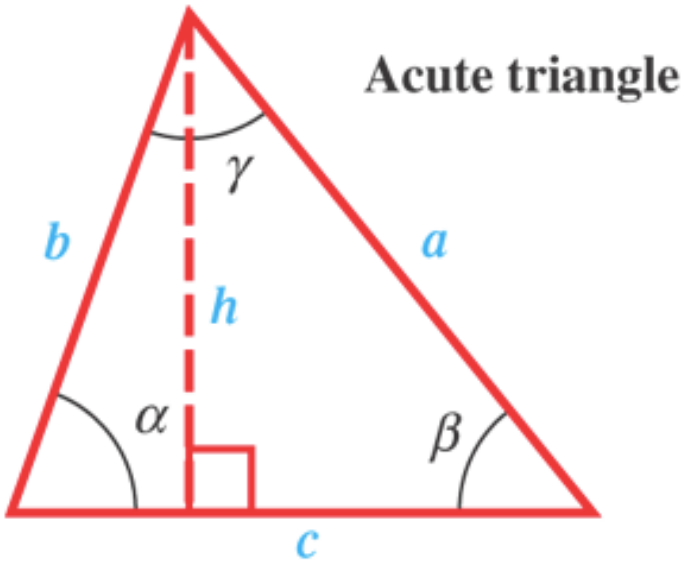
7.1 - Law of Sines

Oblique Triangle: Any triangle that does not have a right angle.

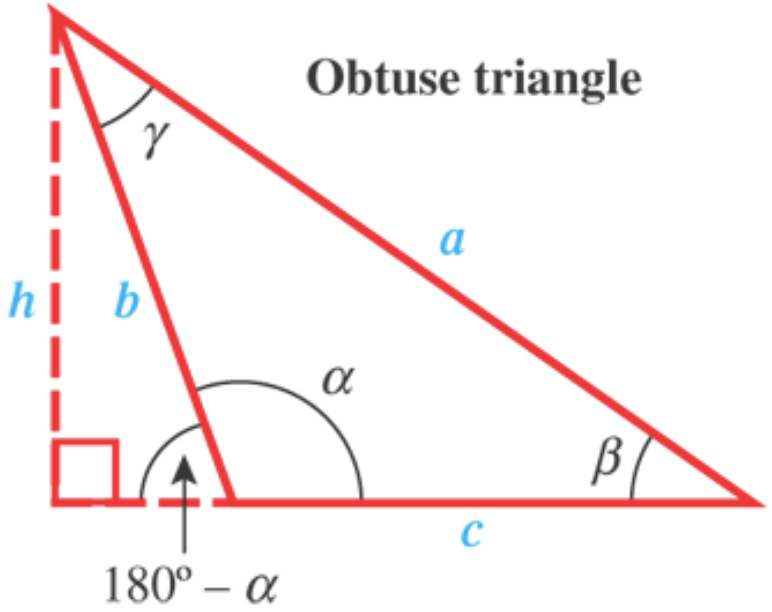


7.1 - Law of Sines

Sine Ratios:



$$\sin\alpha = \frac{h}{b} \qquad \sin\beta = \frac{h}{a}$$



$$\sin(180^\circ - \alpha) = \sin\alpha$$

$$\sin\alpha = \frac{h}{b} \qquad \sin\beta = \frac{h}{a}$$

SAME AS ACUTE!!!

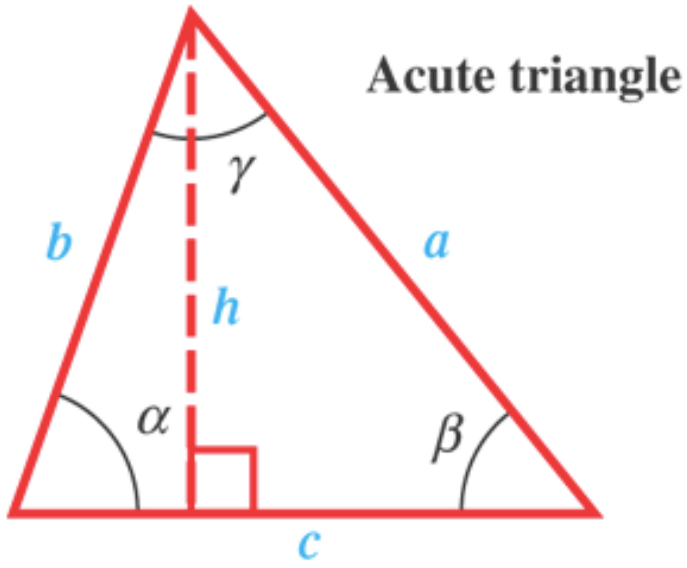
7.1 - Law of Sines

$$\sin\alpha = \frac{h}{b} \qquad \sin\beta = \frac{h}{a}$$

$$h = b\sin\alpha \qquad h = a\sin\beta$$

$$b\sin\alpha = a\sin\beta$$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

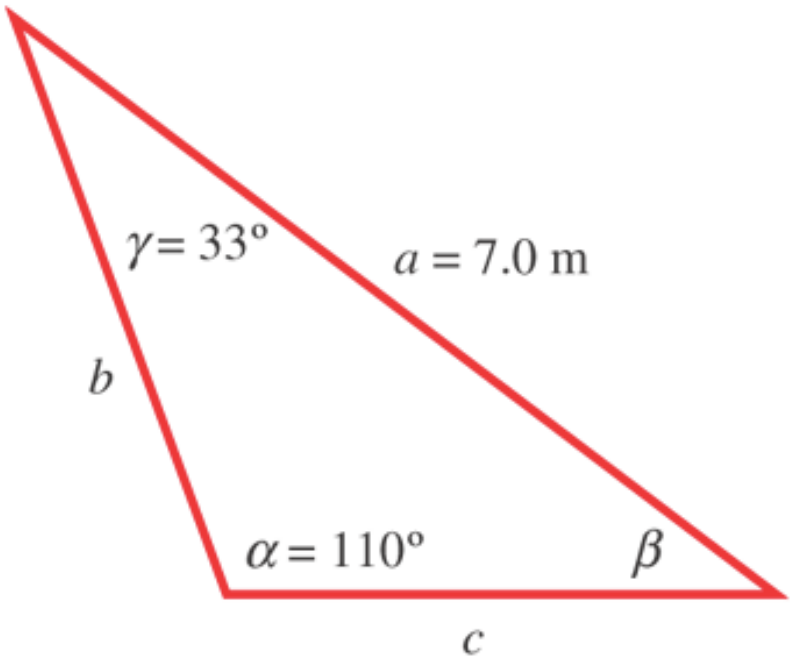


$$\frac{\sin\alpha}{\mathbf{a}} = \frac{\sin\beta}{\mathbf{b}} = \frac{\sin\gamma}{\mathbf{c}}$$

Law of Sines

7.1 - Law of Sines

Solve the triangle (AAS)



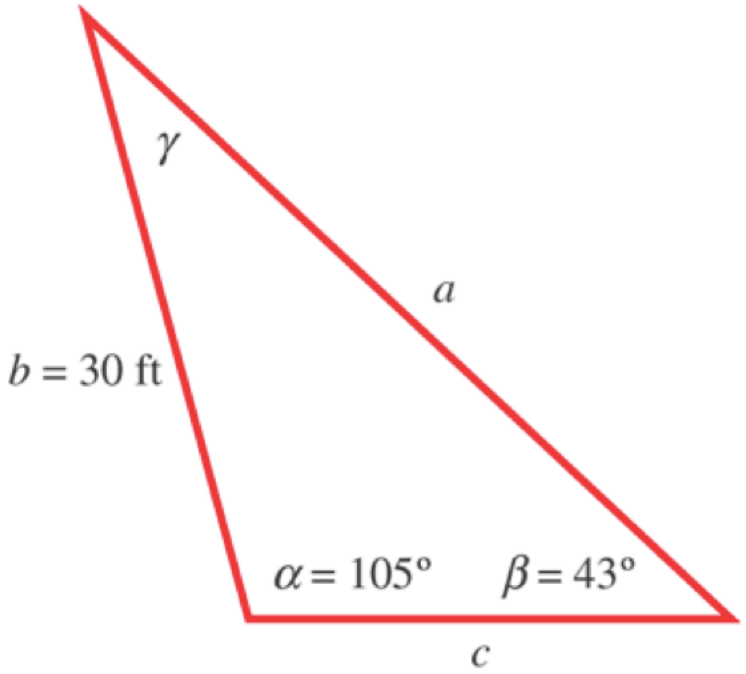
$\beta = 37°$

$b \approx 4.483\ m$

$c \approx 4.057\ m$

7.1 - Law of Sines

Practice: Solve the triangle (AAS)



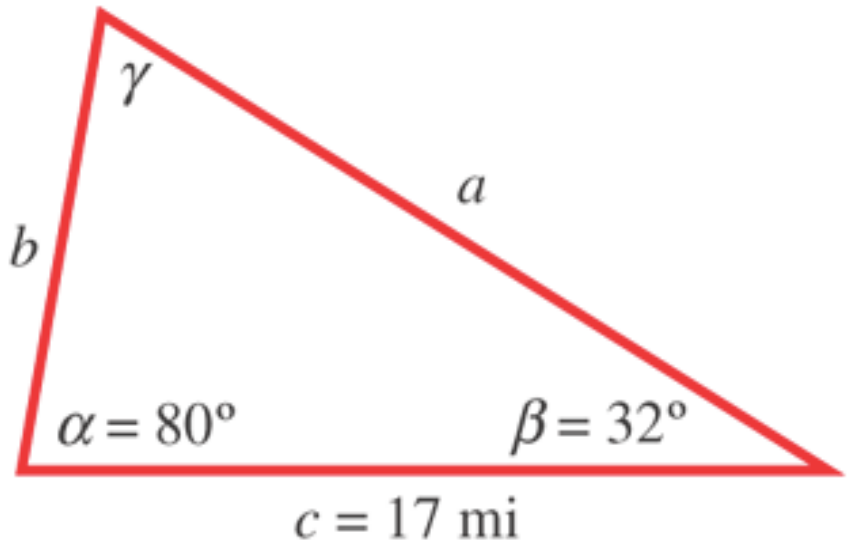
$\gamma = 32^\circ$

$a \approx 42.4895\ ft$

$c \approx 23.3103\ ft$

7.1 - Law of Sines

Solve the triangle (ASA)



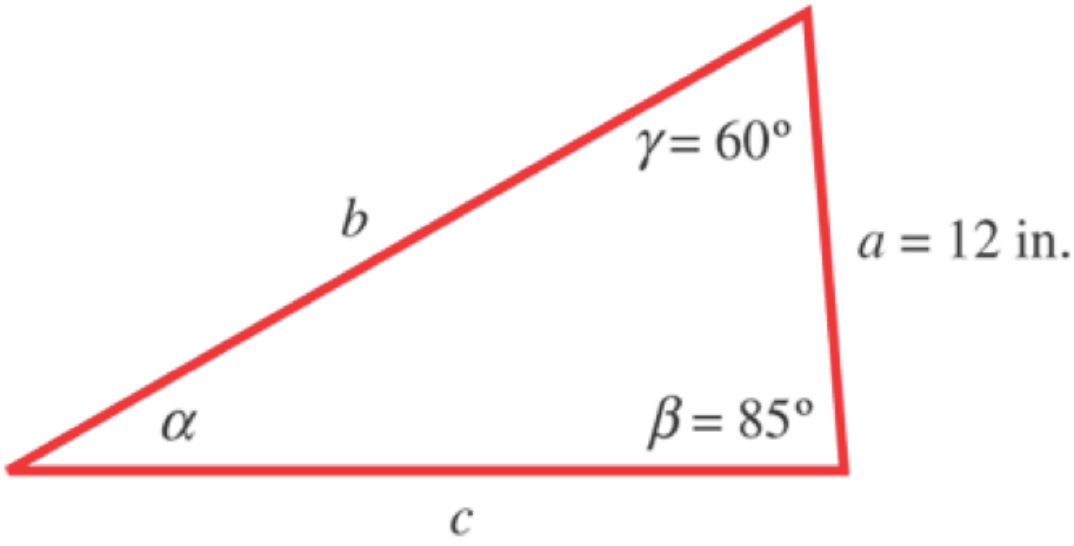
$\gamma = 68^\circ$

$b \approx 9.7161\ mi$

$a \approx 18.0565\ mi$

7.1 - Law of Sines

Practice: Solve the triangle (ASA)



$\alpha = 35^\circ$

$b \approx 20.8417\ in$

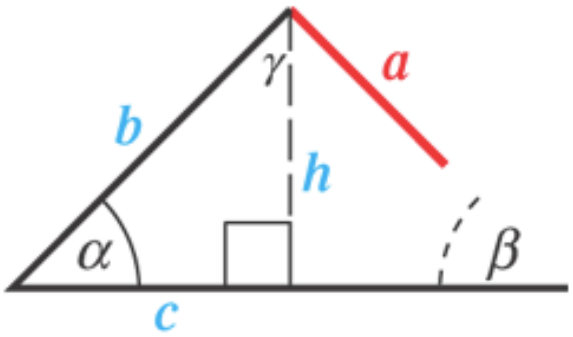
$c \approx 18.1184\ in$

7.1 - Law of Sines

SSA will have one of three solutions given a, b, and α.

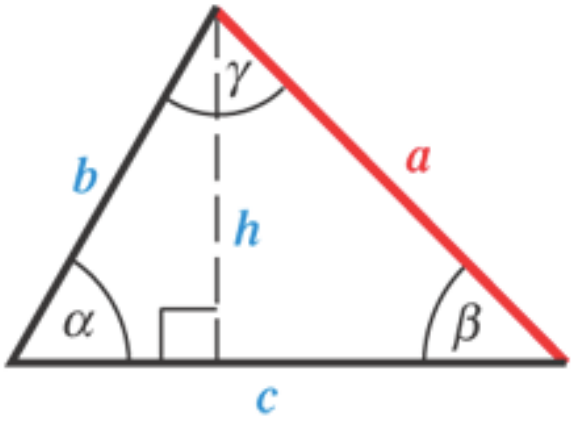
1. **No solution** when the height h is greater than the opposite side a.

$$h = b \sin \alpha \qquad h > a$$



2. **One solution** when side a is greater than or equal to side b.

$$a \geq b$$



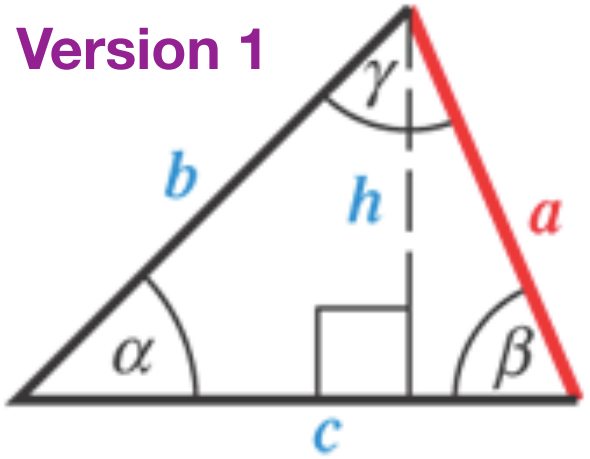
7.1 - Law of Sines

SSA will have one of three solutions given a, b, and α.

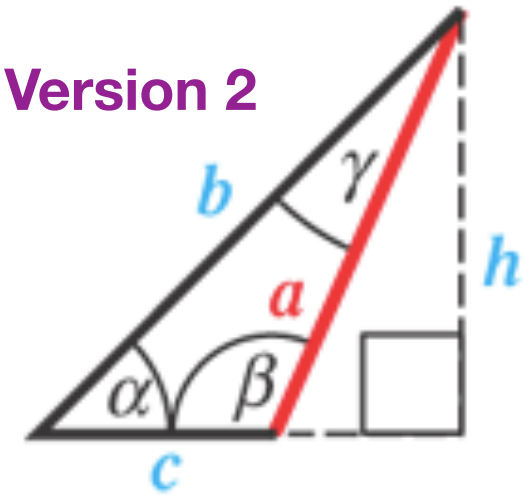
3. **Two solutions** when a is greater than the height h and less than b. (Ambiguous case)

$$h < a < b$$

Version 1



Version 2



7.1 - Law of Sines

(SSA) Solve the triangle $a = 23$ feet, $b = 11$ feet, and $\alpha = 122°$.

$a > b$ One solution

$\beta \approx 23.928°$

$\gamma \approx 34.072°$

$c \approx 15.194\ ft$

Practice: Solve the triangle $\alpha = 133°$, $a = 48$ millimeters, and $c = 17$ millimeters.

$\beta \approx 31.99°$

$\gamma \approx 15.01°$

$b \approx 34.77\ mm$

7.1 - Law of Sines

(SSA) Solve the triangle $a = 8.1$ meters, $b = 8.3$ meters, and $\alpha = 72^\circ$.

$$h = 8.3 \sin 72^\circ \approx 7.89$$

$h < a < b$ Two solutions

$\beta_1 \approx 77.04^\circ$ $\beta_2 \approx 102.96^\circ$

$\gamma_1 \approx 30.96^\circ$ $\gamma_2 \approx 5.04^\circ$

$c_1 \approx 4.38\ m$ $c_2 \approx 0.75\ m$

7.1 - Law of Sines

(SSA) Solve the triangle $a = 6$, $b = 8$, and $\alpha = 107°$.

$$h = 8 \sin 107° \approx 7.65$$

$$a < h$$

No triangle!!!